

PhD Preliminary Written Test

Instructions: For each question, choose the right answer from (a),(b),(c),(d) and mark it on the answer sheet on the last page (page 4). Each correct answer carries a mark of **+1** and each wrong answer carries a negative mark of **-1/4**.

Time: 1 hr

Section A: Objective questions

Q1. What is the value of the expression $\sum_{k=1}^n k2^k$?

- (a) $(n-1)2^{n+1} + 2$ (b) $(2n-1)^2 + 1$
(c) $n2^{n+1} - 2$ (d) $(n+1)2^n - 2$

Q2. Let $n \geq 3$ be a positive integer. How many triples (x, y, z) of positive integers are there such that $x + y + z = n$?

- (a) $\frac{(2n-1)^2 + 1}{2}$ (b) $\frac{(n-1)(n-2)}{2}$
(c) $\frac{n^2 - 2n - 1}{2}$ (d) $\frac{n^2 - 3n + 2}{2}$

Q3. Suppose that a random number generator produces a random element of $\{1, 2, \dots, 9\}$, choosing each element with the same probability. Suppose that n random elements are chosen this way. What is the probability that the product of the n numbers is not divisible by 3?

- (a) $\frac{2^n}{3^n}$ (b) $\frac{4^n}{9^n}$
(c) $\frac{1}{3^n}$ (d) $\frac{1}{9^n}$

Q4. A coin is biased so that the probability of a head is 0.8 and the probability of a tail is 0.2. Suppose that the coin is tossed 5 times in succession. What is the probability that the number of heads is equal to 2?

- (a) 0.0512 (b) 0.00512
(c) 0.0256 (d) 0.00256

Q5. Including the initial parent process, how many processes are created by the program given below?

```
int main()
{
    fork(); fork(); fork(); fork(); fork();
    return 0;
}
```

- (a) 10 (b) 11
(c) 16 (d) 32

Q6. Euclid's algorithm to find the gcd of two integers a and b is shown below:

```
while (b>0)
begin
    r:=a (mod b)
    a:=b
    b:=r
end
gcd:=a
```

When the above algorithm is used to find the greatest common divisor of 180 and 48, what is the sequence of values of r which is computed?

- (a) 132, 84, 36, 12, 0 (b) 96, 48, 0
(c) 36, 6, 0 (d) 36, 12, 0

Q7. How many zeroes are present at the end of the decimal representation of $n!$ when $n = 2013$?

- (a) 402 (b) 501
(c) 600 (d) 699

Q8. Consider the following statements:

Statement I: DNS employs centralized database for resolving domain names to IP addresses
 Statement II: IPv6 completely addresses IP address shortage in Internet

Which of the above statements is true?

- (a) Only Statement I (b) Only Statement II
 (c) Both Statement I & Statement II (d) Neither Statement I nor Statement II

Q9. Statement I: Multimedia is carried in general over UDP/IP network rather than TCP/IP network. Statement II: UDP employs STOP-and-WAIT flow control mechanism.

Which of the above statements is true?

- (a) Only Statement I (b) Only Statement II
 (c) Both Statement I & Statement II (d) Neither Statement I nor Statement II

Q10. Consider a function $F(n)$ given by the following recurrence relation: $F(0) = F(1) = 1$, $F(n) = F(n - 1) + F(n - 2)$. Which of the following is true about the function $F(n)$?

- (a) $F(n) = O(n^2)$ (b) $F(n) = O(2^n)$
 (c) $2^n = O(F(n))$ (d) $\lim_{n \rightarrow \infty} \frac{F(n)}{2^n} = 2$

Q11. What does the following function, written in the C programming language, compute?

```
int f(int x, unsigned int y)
{
    if( y == 0) return 1;
    else if (y%2 == 0) return f(x, y/2)*f(x, y/2);
    else return x*f(x, y/2)*f(x, y/2);
}
```

- (a) y^x (b) $y \bmod x$
 (c) x^y (d) $x \bmod y$

Q12. Let X be a n element set and Y be a m element set. How many functions are there from X to Y ?

- (a) m^n (b) n^m
 (c) $n(n - 1) \dots (n - m + 1)$ (d) $(n!)^m$

Answer Sheet: Please enter a tick mark (✓) for ONLY ONE of the 4 choices for each question.

Question Number	(a)	(b)	(c)	(d)
1				
2				
3				
4				
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12				